Chapter 3: Complex Numbers

EXERCISE 3.1 [PAGES 37 - 38]

Exercise 3.1 | Q 1.1 | Page 37
Write the conjugates of the following complex numbers: 3 + i

SOLUTION

Conjugate of (3 + i) is (3 – i)

Exercise 3.1 | Q 1.2 | Page 37
Write the conjugates of the following complex numbers: 3 – i

Conjugate of (3 – i) is (3 + i)

Exercise 3.1 | Q 1.3 | Page 37
Write the conjugates of the following complex numbers: - √5 - √7 i

SOLUTION

Conjugate of $\left(-\sqrt{5}-\sqrt{7}\,\mathrm{i}\right)\mathrm{is}\left(-\sqrt{5}+\sqrt{7}\,\mathrm{i}\right)$

Exercise 3.1 | Q 1.4 | Page 37
Write the conjugates of the following complex numbers: - √-5

SOLUTION

$$-\sqrt{-5} = -\sqrt{5} \times \sqrt{-1} = \sqrt{-5}\,\mathrm{i}$$

Exercise 3.1 | Q 1.5 | Page 37
Write the conjugates of the following complex numbers: 5i

SOLUTION

Conjugate of 5i is – 5i

Exercise 3.1 | Q 1.6 | Page 37
Write the conjugates of the following complex numbers: √5 – i

SOLUTION

Conjugate of $\sqrt{5} - \mathrm{i}$ is $\sqrt{5} + \mathrm{i}$

Exercise 3.1 | Q 1.7 | Page 37
Write the conjugates of the following complex numbers: √2 + √3 i

SOLUTION

Conjugate of $\sqrt{2}+\sqrt{3}\,\mathrm{i}$ is $\sqrt{2}-\sqrt{3}\,\mathrm{i}$

Exercise 3.1 | Q 2.1 | Page 37

Express the following in the form of a + ib, a, b ∈ R, i = √-1. State the values of a and b:
(1 + 2i)(– 2 + i)

SOLUTION

$(1 + 2i)(-2 + i) = -2 + i - 4i + 2i^2$
$= -2 - 3i + 2(-1)$ ...[∵ $i^2 = -1$]
∴ $(1 + 2i)(-2 + i) = -4 - 3i$
∴ a = – 4 and b = – 3

Exercise 3.1 | Q 2.2 | Page 37

Express the following in the form of a + ib, a, b ∈ R, i = √-1. State the values of a and b:

$$\frac{\mathrm{i}(4+3\mathrm{i})}{1-\mathrm{i}}$$

SOLUTION

$$\frac{\mathrm{i}(4+3\mathrm{i})}{1-\mathrm{i}}=\frac{4\mathrm{i}+3\mathrm{i}^2}{1-\mathrm{i}}$$

$$=\frac{-3+4\mathrm{i}}{1-\mathrm{i}} \quad ...[\because i^2 = -1]$$

$$= \frac{(-3 \ + 4i)(1+i)}{(1-i)(1+i)}$$

$$= \frac{3 - 3i + 4i + 4i^2}{1 - i^2}$$

$$= \frac{-3 + i + 4(-1)}{1-(-1)}$$ ...[∵ $i^2 = -1$]

$$= \frac{-7 + i}{2}$$

$$\therefore \frac{i(4+3i)}{1-i} = \frac{-7}{2} + \frac{1}{2}i$$

$$\therefore a = \frac{-7}{2} \text{ and } b = \frac{1}{2}.$$

Exercise 3.1 | Q 2.3 | Page 37

Express the following in the form of a + ib, a, b ∈ R, i = √-1. State the values of a and b:

$$\frac{(2+i)}{(3-i)(1+2i)}$$

SOLUTION

$$\frac{(2+i)}{(3-i)(1+2i)} = \frac{2+i}{3+6i-i-2i^2}$$

$$= \frac{2+i}{3+5i-2(-1)}$$ ...[∵ $i^2 = -1$]

$$= \frac{2+i}{5+5i}$$

$$= \frac{2+i}{5(1+i)} = \frac{(2+i)(1-i)}{5(1+i)(1-i)}$$

$$= \frac{2-2i+i-i^2}{5(1-i^2)}$$

$= \frac{2 - i - (-1)}{5[1 - (-1)]}$ ...[∵ $i^2 = -1$]

$= \frac{3 - i}{10}$

$\therefore \frac{2 + i}{(3 - i)(1 + 2i)} = \frac{3}{10} - \frac{1}{10}i$

$\therefore a = \frac{3}{10}$ and $b = \frac{-1}{10}$.

Exercise 3.1 | Q 2.4 | Page 37

Express the following in the form of a + ib, a, b ∈ R, i = √-1. State the values of a and b:

$\frac{3 + 2i}{2 - 5i} + \frac{3 - 2i}{2 + 5i}$

SOLUTION

$\frac{3 + 2i}{2 - 5i} + \frac{3 - 2i}{2 + 5i}$

$= \frac{(3 + 2i)(2 + 5i) + (2 - 5i)(3 - 2i)}{(2 - 5i)(2 + 5i)}$

$= \frac{6 + 15i + 4i + 10i^2 + 6 - 4i - 15i + 10i^2}{4 - 25i^2}$

$= \frac{12 + 20i^2}{4 - 25i^2}$

$= \frac{12 + 20(-1)}{4 - 25(-1)}$ ...[∵ $i^2 = -1$]

$= \frac{-8}{29}$

$\therefore \frac{3 + 2i}{2 - 5i} + \frac{3 - 2i}{2 + 5i} = \frac{-8}{29} + 0i$

$\therefore a = \frac{-8}{29}$ and $b = 0$

Exercise 3.1 | Q 2.5 | Page 37

Express the following in the form of a + ib, a, b ∈ R, i = √-1. State the values of a and b:

$$\frac{2+\sqrt{-3}}{4+\sqrt{-3}}$$

SOLUTION

$$\frac{2+\sqrt{-3}}{4+\sqrt{-3}} = \frac{2+\sqrt{3}i}{4+\sqrt{3}i}$$

$$= \frac{\left(2+\sqrt{3}i\right)\left(4-\sqrt{3}i\right)}{\left(4+\sqrt{3}i\right)\left(4-\sqrt{3}i\right)}$$

$$= \frac{8-2\sqrt{3}i+4\sqrt{3}i-3i^2}{16-3i^2}$$

$$= \frac{8+2\sqrt{3}i-3(-1)}{16-3(-1)} \quad ...[\because i^2 = -1]$$

$$= \frac{11+2\sqrt{3}i}{19}$$

$$\therefore \frac{2+\sqrt{-3}}{4+\sqrt{-3}} = \frac{11}{19} + \frac{2\sqrt{3}}{19}i$$

$$\therefore a = \frac{11}{19} \text{ and } b = \frac{2\sqrt{3}}{19}$$

Exercise 3.1 | Q 2.6 | Page 37

Express the following in the form of a + ib, a, b ∈ R, i = √-1. State the values of a and b:

(2 + 3i)(2 – 3i)

SOLUTION

$(2 + 3i)(2 - 3i) = 4 - 9i^2$

$= 4 - 9(-1) \quad ...[\because i^2 = -1]$

$= 4 + 9 = 13$

$\therefore (2 + 3i)(2 - 3i) = 13 + 0i$

$\therefore a = 13$ and $b = 0$

Exercise 3.1 | Q 2.7 | Page 38

Express the following in the form of a + ib, a, b ∈ R, i = √-1. State the values of a and b:

$$\frac{4i^8 - 3i + 3}{3i^{11} - 4i^{10} - 2}$$

SOLUTION

$$\frac{4i^8 - 3i + 3}{3i^{11} - 4i^{10} - 2} = \frac{4\left(i^4\right)^2 - 3\left(i^4\right)^2 \cdot i + 3}{3\left(i^4\right)^2 \cdot i^3 - 4\left(i^4\right)^2 \cdot i^2 - 2}$$

Since, $i^2 = -1$, $i^3 = -i$ and $i^4 = 1$

$$\therefore \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2} = \frac{4(1)^2 - 3(1)^2 \cdot i + 3}{3(1)^2(-i) - 4(1)^2(-1) - 2}$$

$$= \frac{4 - 3i + 3}{-3i + 4 - 2}$$

$$= \frac{7 - 3i}{2 - 3i}$$

$$= \frac{(7 - 3i)(2 - 3i)}{(2 - 3i)(2 + 3i)}$$

$$= \frac{14 + 21i - 6i - 9i^2}{4 - 9(-1)}$$

$$= \frac{14 + 15i - 9(-1)}{4 - 9(-1)}$$

$$= \frac{23 + 15i}{13}$$

$$\therefore \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2} = \frac{23}{13} + \frac{15}{13}i$$

$$\therefore a = \frac{23}{13} \text{ and } b = \frac{15}{13}$$

Exercise 3.1 | Q 3 | Page 38

Show that $\left(-1+\sqrt{3}i\right)^3$ is a real number.

SOLUTION

$\left(-1+\sqrt{3}i\right)^3$

$= (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3$ ...$[(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3]$

$= -1 + 3\sqrt{3}i - 3\left(3i^2\right) + 3\sqrt{3}i^3$

$= -1 + 3\sqrt{3}i - 3\left(3i^2\right) + 3\sqrt{3}i$ ...$[\because i^2 = -1, i^3 = -i]$

$= -1 + 9$

$= 8$, which is a real number.

Exercise 3.1 | Q 4.1 | Page 38

Evaluate the following: i^{35}

SOLUTION

We know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$i^{35} = (i^4)^8 (i^2)i = (1)^8 (-1)i = -i$

Exercise 3.1 | Q 4.2 | Page 38

Evaluate the following: i^{888}

SOLUTION

We know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$i^{888} = (i^4)^{222} = (1)^{222} = 1$

Exercise 3.1 | Q 4.3 | Page 38

Evaluate the following: i^{93}

SOLUTION

We know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$i^{93} = (i^4)^{23}.i = (1)^{23}.i = i$

Exercise 3.1 | Q 4.4 | Page 38

Evaluate the following: i^{116}

SOLUTION

We know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$i^{116} = (i^4)^{29} = (1)^{29} = 1$

Exercise 3.1 | Q 4.5 | Page 38

Evaluate the following: i^{403}

SOLUTION

We know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$i^{403} = (i^4)^{100} (i^2)i = (1)^{100} (-1)i = -i$

Exercise 3.1 | Q 4.6 | Page 38

Evaluate the following: $\dfrac{1}{i^{58}}$

SOLUTION

We know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$$\frac{1}{i^{58}} = \frac{1}{(i^4)^{14} \cdot i^2} = \frac{1}{(1)^{14}(-1)} = -1$$

Exercise 3.1 | Q 4.7 | Page 38

Evaluate the following: $i^{30} + i^{40} + i^{50} + i^{60}$

SOLUTION

We know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$= (i^4)^7\, i^2 + (i^4)^{10} + (i^4)^{12}\, i^2 + (i^4)^{15}$

$= (1)^7 (-1) + (1) + (1)^{10} + (1)^{12} (-1) + (1)^{15}$

$= -1 + 1 - 1 + 1$

$= 0.$

Exercise 3.1 | Q 5 | Page 38

Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number.

SOLUTION

$1 + i^{10} + i^{20} + i^{30}$

$= 1 + (i^4)^2 .i^2 + (i^4)^5 + (i^4)^7 .i^2$

$= 1 + (1)^2 (-1) + (1)^5 + (1)^7 (-1)$ ...[$\because$ $i^4 = 1, i^2 = -1$]

$= 1 - 1 + 1 - 1$

$= 0$, which is a real number.

Exercise 3.1 | Q 6.1 | Page 38

Find the value of $i^{49} + i^{68} + i^{89} + i^{110}$

SOLUTION

$i^{49} + i^{68} + i^{89} + i^{110}$

$= (i^4)^{12} .i + (i^4)^{17} + (i^4)^{22} .i + (i^4)^{27} .i^2$

$= (1)^{12} .i + (1)^{17} + (1)^{22} .i + (1)^{27} (-1)$ ...[$\because$ $i^4 = 1, i^2 = -1$]

$= i + 1 + i - 1$
$= 2i$

Exercise 3.1 | Q 6.2 | Page 38
Find the value of $i + i^2 + i^3 + i^4$

SOLUTION

$i + i^2 + i^3 + i^4$
$= i + i^2 + i^2 .i + i^4$
$= i - 1 - i + 1$ ...[$\because$ $i^2 = -1, i^4 = 1$]
$= 0.$

Exercise 3.1 | Q 7 | Page 38
Find the value of $1 + i^2 + i^4 + i^6 + i^8 + ... + i^{20}$.

SOLUTION

$1 + i^2 + i^4 + i^6 + i^8 + ... + i^{20}$
$= 1 + (i^2 + i^4) + (i^6 + i^8) + (i^{10} + i^{12}) + (i^{14} + i^{16}) + (i^{18} + i^{20})$
$= 1 + [i^2 + (i^2)^2] + [(i^2)^3 + (i^2)^4] + [(i^2)^5 + (i^2)^6] + [(i^2)^7 + (i^2)^8] + [(i^2)^9 + (i^2)^{10}]$
$= 1 + [-1 + (-1)^2] + [(-1)^3 + (-1)^4] + [(-1)^5 + (-1)^6] + [(-1)^7 + (-1)^8] + [(-1)^9 + (-1)^{10}]$ ...[$\because$ $i^2 = -1$]
$= 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + (-1 + 1) + (-1 + 1)$
$= 1 + 0 + 0 + 0 + 0 + 0$
$= 1.$

Exercise 3.1 | Q 8.1 | Page 38
Find the values of x and y which satisfy the following equations ($x, y \in R$): $(x + 2y) + (2x - 3y i + 4i = 5$

SOLUTION

$(x + 2y) + (2x - 3y i + 4i = 5$
$\therefore (x + 2y) + (2x - 3y)i = 5 - 4i$
Equating real and imaginary parts, we get
$x + 2y = 5$ ...(i)
and $2x - 3y = -4$ ...(ii)
Equation (i) x 2 – equation (ii) gives
$7y = 14$
$\therefore y = 2$
Putting $y = 2$ in (i), we get
$x + 2(2) = 5$
$\therefore x + 4 = 5$
$\therefore x = 1$
$\therefore x = 1$ and $y = 2$.

Exercise 3.1 | Q 8.2 | Page 38

Find the values of x and y which satisfy the following equations ($x, y \in R$):

$$\frac{x+1}{1+i} + \frac{y-1}{1-i} = i$$

SOLUTION

$$\frac{x+1}{1+i} + \frac{y-1}{1-i} = i$$

$$\therefore \frac{(x+1)(1-i)+(y-1)(1+i)}{(1+i)(1-i)} = i$$

$$\therefore \frac{x-xi+1-i+y+yi-1-i}{1-i^2}$$

$$\therefore \frac{(x+y)+(y-x-2)i}{1-(-1)} = i \quad ...[\because i^2 = -1]$$

$\therefore (x + y) + (y - x - 2)i = 2i$
$\therefore (x + y) + (y - x - 2)i = 0 + 2i$
Equating real and imaginary parts, we get
$x + y = 0$ and $y - x - 2 = 2$
$\therefore x + y = 0$ ...(i)
and $-x + y = 4$ ...(ii)
Adding (i) and (ii), we get
$2y = 4$
$\therefore y = 2$
Putting $y = 2$ in (i), we get
$x + 2 = 0$
$\therefore x = -2$
$\therefore x = -2$ and $y = 2$.

Exercise 3.1 | Q 9.1 | Page 38

Find the value of: $x^3 - x^2 + x + 46$, if $x = 2 + 3i$

SOLUTION

$x = 2 + 3i$
$\therefore x - 2 = 3i$
$\therefore (x - 2)^2 = 9i^2$
$\therefore x^2 - 4x + 4 = 9(-1)$ ...[$\because i^2 = -1$]
$\therefore x^2 - 4x + 13 = 0$ ...(i)

$$
\begin{array}{r}
x + 3 \\
x^2 - 4x + 13\overline{)x^3 + x^2 + x + 46} \\
x^3 - 4x^2 + 13x \\
- \quad + \quad - \\
\hline
3x^2 - 12x + 46 \\
3x^2 - 12x + 39 \\
- \quad + \quad - \\
\hline
7
\end{array}
$$

$\therefore x^3 - x^2 + x + 46$

$= (x^2 - 4x + 13)(x + 3) + 7$

$= 0(x + 3) + 7$ ...[From (i)]

$= 7.$

Exercise 3.1 | Q 9.2 | Page 38

Find the value of: $2x^3 - 11x^2 + 44x + 27$, if $x = \dfrac{25}{3 - 4i}$

SOLUTION

$x = \dfrac{25}{3 - 4i}$

$\therefore x = \dfrac{25(3 + 4i)}{(3 - 4i)(3 + 4i)}$

$= \dfrac{25(3 + 4i)}{9 - 16i^2}$

$= \dfrac{25(3 + 4i)}{9 - 16(-1)}$ ...[$\because i^2 = -1$]

$= \dfrac{25(3 + 4i)}{25}$

$\therefore x = 3 + 4$

$\therefore x - 3 = 4i$

$\therefore (x - 3)^2 = 16i^2$

$\therefore x^2 - 6x + 9 = 16(-1)$ ...[$\because i^2 = -1$]

$\therefore x^2 - 6x + 25 = 0$

$$\begin{array}{r|l} & 2x + 1 \\ \hline x^2 - 6x + 25 & 2x^3 - 11x^2 + 44x + 27 \\ & 2x^3 - 12x^2 + 50x \\ & - \quad + \quad - \\ \hline & x^2 - 6x + 27 \\ & x^2 - 6x + 25 \\ & - \quad + \quad - \\ \hline & 2 \end{array}$$

EXERCISE 3.2 [PAGE 40]

Exercise 3.2 | Q 1.1 | Page 40

Find the square root of the following complex numbers: $-8 - 6i$

SOLUTION

Let $\sqrt{-8 - 6\mathbf{i}} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$-8 - 6i = (a + bi)^2$

$\therefore -8 - 6i = a^2 + b^2i^2 + 2abi$

$\therefore -8 - 6i = (a^2 - b^2) + 2abi$ ...[$\because i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = -8$ and $2ab = -6$

$\therefore a^2 - b^2 = -8$ and $b = \dfrac{-3}{a}$

$\therefore a^2 - \left(-\frac{3}{a}\right)^2 = -8$

$\therefore a^2 - \frac{9}{a^2} = -8$

$\therefore a^4 - 9 = -8a^2$

$\therefore a^4 + 8a^2 - 9 = 0$

$\therefore (a^2 + 9)(a^2 - 1) = 0$

$\therefore a^2 = -9$ or $a^2 = 1$

But $a \in R$

$\therefore a^2 \neq -9$

$\therefore a^2 = 1$

$\therefore a = \pm 1$

when a = 1, $b = \frac{-3}{1} = -3$

when a = − 1, $b = \frac{-3}{-1} = 3$

$\therefore \sqrt{-8 - 6i} = \pm (1 - 3i)$.

Exercise 3.2 | Q 1.2 | Page 40

Find the square root of the following complex numbers: 7 + 24i

SOLUTION

Let $\sqrt{7 + 24i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$7 + 24i = (a + bi)^2$

$\therefore 7 + 24i = a^2 + b^2i^2 + 2abi$

$\therefore 7 + 24i = (a^2 - b^2) + 2abi$ ...[$\because i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 7$ and $2ab = 24$

$\therefore a^2 - b^2 = 7$ and $b = \dfrac{12}{a}$

$\therefore a^2 - \left(\dfrac{12}{a}\right)^2 = 7$

$\therefore a^2 - \dfrac{144}{a^2} = 7$

$\therefore a^4 - 144 = 7a^2$

$\therefore a^4 - 7a^2 - 144 = 0$

$\therefore (a^2 - 16)(a^2 + 9) = 0$

$\therefore a^2 = 16$ or $a^2 = -9$

But $a \in R$

$\therefore a^2 \neq -9$

$\therefore a^2 = 16$

$\therefore a = \pm 4$

When $a = 4$, $b = \dfrac{12}{4} = 3$

When $a = -4$, $b = \dfrac{12}{-4} = -3$

$\therefore \sqrt{7 + 24i} = \pm (4 + 3i)$.

Exercise 3.2 | Q 1.3 | Page 40

Find the square root of the following complex numbers: $1 + 4\sqrt{3}i$

SOLUTION

Let $\sqrt{1 + 4\sqrt{3}i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$1 + 4\sqrt{3}i = (a + bi)^2$

$\therefore 1 + 4\sqrt{3}\,i = a^2 + b^2i^2 + 2abi$

$\therefore 1 + 4\sqrt{3}\,i = (a^2 - b^2) + 2abi \quad ...[\because i^2 = -1]$

Equating real and imaginary parts, we get

$a^2 - b^2 = 1$ and $2ab = 4\sqrt{3}$

$\therefore a^2 - b^2 = 1$ and $b = \frac{2\sqrt{3}}{a}$

$\therefore a^2 - \left(\frac{2\sqrt{3}}{a}\right)^2 = 1$

$\therefore a^2 - \frac{12}{a^2} = 1$

$\therefore a^4 - 12 = a^2$

$\therefore a^4 - a^2 - 12 = 0$

$\therefore (a^2 - 4)(a^2 + 3) = 0$

$\therefore a^2 = 4$ or $a^2 = -3$

But $a \in R$

$\therefore a^2 \neq -3$

$\therefore a^2 = 4$

$\therefore a = \pm 2$

When $a = 2$, $b = \frac{2\sqrt{3}}{2} = \sqrt{3}$

When $a = -2$, $b = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\therefore \sqrt{1 + 4\sqrt{3}i} = \pm\left(2 + \sqrt{3}i\right)$

Exercise 3.2 | Q 1.4 | Page 40

Find the square root of the following complex numbers: $3 + 2\sqrt{10}\,i$

SOLUTION

Let $\sqrt{3+2\sqrt{10}\mathrm{i}}$ = a + bi, where a, b ∈ R

Squaring on both sides, we get

$3+2\sqrt{10}\,\mathrm{i}$ = $(a + bi)^2$

∴ $3+2\sqrt{10}\,\mathrm{i}$ = $a^2 + b^2i^2 + 2abi$

3 + 2 sqrt(10) "i"` = $(a^2 - b^2) + 2abi$ ...[∵ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 3$ and $2ab = 2\sqrt{10}$

∴ $a^2 - b^2 = 3$ and $b = \frac{\sqrt{10}}{a}$

∴ $a^2-\left(\frac{\sqrt{10}}{a}\right)^2 = 3$

∴ $a^2-\frac{10}{a^2} = 3$

∴ $a^4 - 10 = 3a^2$

∴ $a^4 - 3a^2 - 10 = 0$

∴ $(a^2 - 3a^2 - 10 = 0$

∴ $a^2 = 5$ or $a^2 = -2$

But a ∈ R

∴ $a^2 \neq -2$

∴ $a^2 = 5$

∴ $a = \pm\sqrt{5}$

When a = $\sqrt{5}, b=\frac{\sqrt{10}}{\sqrt{5}}=\sqrt{2}$

When a = $-\sqrt{5}, \mathrm{b}=\dfrac{\sqrt{10}}{-\sqrt{5}}$ $-\sqrt{2}$

$\therefore \sqrt{3+2\sqrt{10}\mathrm{i}}=\pm\left(\sqrt{5}+\sqrt{2}\mathrm{i}\right)$

Exercise 3.2 | Q 1.5 | Page 40

Find the square root of the following complex numbers: $2\left(1-\sqrt{3}\,\mathrm{i}\right)$

SOLUTION

Let $\sqrt{2\left(1-\sqrt{3}\mathrm{i}\right)}$ = a + bi, where a, b ∈ R

Squaring on both sides, we get

$2\left(1-\sqrt{3}\mathrm{i}\right)$ = $(a + bi)^2$

$\therefore 2\left(1-\sqrt{3}\mathrm{i}\right)$ = $a^2 + b^2i^2 + 2abi$

$\therefore 2-2\sqrt{3}\mathrm{i}$ = $(a^2 - b^2) + 2abi$ ...[∵ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 2$ and $2ab = -2\sqrt{3}$

∴ $a^2 - b^2 = 2$ and $b = -\dfrac{\sqrt{3}}{a}$

$\therefore a^2 - \dfrac{3}{a^2} = 2$

∴ $a^4 - 3 = 2a^2$

∴ $a^4 - 2a^2 - 3 = 0$

∴ $(a^2 - 3)(a^2 + 1) = 0$

∴ $a^2 = 3$ or $a^2 = -1$

But a ∈ R

∴ $a^2 \neq -1$

∴ $a^2 = 3$

$\therefore$ a = $\pm\sqrt{3}$

When a = $\sqrt{3}, b = \frac{-\sqrt{3}}{\sqrt{3}}$ = − 1

When a = $\sqrt{3}, \mathrm{b} = \frac{-\sqrt{3}}{-\sqrt{3}}$ = 1

$\therefore \sqrt{2\left(1-\sqrt{3}\mathrm{i}\right)} = \pm\left(\sqrt{3}-\mathrm{i}\right).$

Exercise 3.2 | Q 2.1 | Page 40

Solve the following quadratic equation: $8x^2 + 2x + 1 = 0$

SOLUTION

Given equation is $8x^2 + 2x + 1 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 8, b = 2, c = 1
Discriminant = $b^2 - 4ac$
= $(2)^2 - 4 \times 8 \times 1$
= 4 – 32
– 28 < 0
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{a}}$$

$$= \frac{-2 - \sqrt{-28}}{2(8)}$$

$$= \frac{-2 \pm 2\sqrt{7}\mathrm{i}}{16}$$

$$\therefore x = \frac{-1 \pm \sqrt{7}\mathrm{i}}{8}$$

∴ the roots of the given equation are

$$\frac{-1+\sqrt{7}\mathrm{i}}{8} \text{ and } \frac{-1-\sqrt{7}\mathrm{i}}{8}.$$

Exercise 3.2 | Q 2.2 | Page 40

Solve the following quadratic equation: $2x^2 - \sqrt{3}\,x + 1 = 0$

SOLUTION

Given equation is $2x^2 - \sqrt{3}\,x + 1 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -\sqrt{3}$, $c = 1$

Discriminant = $b^2 - 4ac$

$= \left(-\sqrt{3}\right)^2 - 4 \times 2 \times 1$

$= 3 - 8$

$= -5 < 0$

So, the given equation has complex roots.

These roots are given by

$$x = \frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{ac}}$$

$$= \frac{--\sqrt{3} + \sqrt{-5}}{2\ 2}$$

$$\therefore x = \frac{\sqrt{3} \pm \sqrt{5}\mathrm{i}}{4}$$

∴ the roots of the given equation are

$$\frac{\sqrt{3}+\sqrt{5}\mathrm{i}}{4} \text{ and } \frac{\sqrt{3}-\sqrt{5}\mathrm{i}}{4}.$$

Exercise 3.2 | Q 2.3 | Page 40

Solve the following quadratic equation: $3x^2 - 7x + 5 = 0$

SOLUTION

Given equation is $3x^2 – 7x + 5 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 3, b = – 7, c = 5
Discriminant = $b^2 – 4ac$
$= (– 7)^2 – 4 \times 3 \times 5$
= 49 – 60
= – 11 < 0
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{- - 7 \pm \sqrt{-11}}{2\ 3}$$

$$\therefore x = \frac{7 \pm \sqrt{11}i}{6}$$

$\therefore$ the roots of the given equation are $\frac{7 \pm \sqrt{11}i}{6}$ and $\frac{7 - \sqrt{11}i}{6}$.

Exercise 3.2 | Q 2.4 | Page 40
Solve the following quadratic equation: $x^2 – 4x + 13 = 0$

SOLUTION

Given equation is $x^2 – 4x + 13 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a =1, b = 4, c = 13
Discriminant = $b^2 – 4ac$
$= (– 4)^2 – 4 \times 1 \times 13$
= 16 – 52
= – 36 < 0
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{- -4 \pm \sqrt{-36}}{2\ 1}$$

$$= \frac{4 \pm 6i}{2}$$

$= 2 \pm 3i$

∴ the roots of the given equation are $2 \pm 3i$ and $2 - 3i$.

Exercise 3.2 | Q 3.1 | Page 40

Solve the following quadratic equation: $x^2 + 3ix + 10 = 0$

SOLUTION

Given equation is $x^2 + 3ix + 10 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = 3i$, $c = 10$
Discriminant $= b^2 - 4ac$
$= (3i)^2 - 4 \times 1 \times 10$
$= 9i^2 - 40$
$= -9 - 40$ ...[∵ $i^2 = -1$]
$= -49 < 0$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3i \pm \sqrt{-49}}{2(1)}$$

$$\therefore x = \frac{-3i + 7i}{2}$$

$$\therefore x = \frac{-3i + 7i}{2} \text{ or } x = \frac{-3i - 7i}{2}$$

∴ x = 2I or x = – 5i

∴ the roots of the given equation are 2i and – 5i.

Exercise 3.2 | Q 3.2 | Page 40

Solve the following quadratic equation: $2x^2 + 3ix + 2 = 0$

SOLUTION

Given equation is $2x^2 + 3ix + 2 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 2, b = 3i, c = 2
Discriminant = b2 – 4ac
$= (3i)^2 - 4 \times 2 \times 2$
$= 9i^2 - 16$
$= -9 - 16$ ...[$\because i^2 = -1$]
$= -25 < 0$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3i \pm \sqrt{-25}}{2(2)}$$

$$\therefore x = \frac{-3i \pm 5i}{4}$$

$$\therefore x = \frac{-3i \pm 5i}{4} \text{ or } x = \frac{-3i - 5i}{4}$$

$$\therefore x = \frac{1}{2}i \text{ or } x = -2i$$

$\therefore$ the roots of the given equation are $\frac{1}{2}i$ and $-2i$.

Exercise 3.2 | Q 3.3 | Page 40

Solve the following quadratic equation: $x^2 + 4ix - 4 = 0$

SOLUTION

Given equation is $x^2 + 4ix - 4 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 1, b = 4i, c = – 4
Discriminant = $b^2 - 4ac$
$= (4i)^2 - 4 \times 1 \times -4$
$= 16i^2 + 16$

$= -16 + 16$ ...[∵ $i^2 = -1$]
$= 0$
So, the given equation has equal roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4i + \sqrt{0}}{2(1)}$$

$$= \frac{-4i}{2}$$

∴ $x = -2i$

∴ the roots of the given equation are $-2i$ and $-2i$.

Exercise 3.2 | Q 3.4 | Page 40

Solve the following quadratic equation: $ix^2 - 4x - 4i = 0$

SOLUTION

$ix^2 - 4x - 4i = 0$
Multiplying throughout by i, we get
$i^2x^2 - 4ix - 4i^2 = 0$
∴ $-x^2 - 4ix + 4 = 0$ ...[∵ $i^2 = -1$]
∴ $x^2 + 4ix - 4 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = 4i$, $c = -4$
Discriminant $= b^2 - 4ac$
$= (4i)^2 - 4 \times 1 - 4$
$= 16i^2 + 16$
$= -16 + 16$ ...[∵ $i^2 = -1$]
$= 0$
So, the given equation has equal roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4i \pm \sqrt{0}}{2(1)}$$

$$= \frac{-4i}{2}$$

$\therefore x = -2i$

$\therefore$ the roots of the given equation are $-2i$ and $-2i$.

Exercise 3.2 | Q 4.1 | Page 40

Solve the following quadratic equation: $x^2 - (2 + i)x - (1 - 7i) = 0$

SOLUTION

Given equation is $x^2 - (2 + i)x - (1 - 7i) = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -(2 + i)$, $c = -(1 - 7i)$
Discriminant $= b^2 - 4ac$
$= [-(2 + i)]^2 - 4 \times 1 - (1 - 7i)$
$= 4 + 4i + i^2 + 4 - 28i$
$= 4 + 4i - 1 + 4 - 28i$ ...[$\because i^2 = -1$]
$= 7 - 24i$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-[-(2 + i)] \pm \sqrt{7 - 24i}}{2(1)}$$

$$= \frac{(2 + i) \pm \sqrt{7 - 24i}}{2}$$

Let $\sqrt{7 - 24i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get
$7 - 24i = a^2 + i^2b^2 + 2abi$
$\therefore 7 - 24i = (a^2 - b^2) + 2abi$ ...[$\because i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 - b^2 = 7$ and $2ab = -24$

$\therefore a^2 - b^2 = 7$ and $b = \frac{-12}{a}$

$\therefore a^2 - \left(\frac{-12}{a}\right)^2 = 7$

$\therefore a^2 - \frac{144}{a^2} = 7$

$\therefore a^4 - 144 = 7a^2$
$\therefore a^4 - 7a^2 - 144 = 0$
$\therefore (a^2 - 16)(a^2 + 9) = 0$
$\therefore a^2 = 16$ or $a^2 = -9$
But $a \in R$
$\therefore a^2 \neq -9$
$\therefore a^2 = 16$
$\therefore a = \pm 4$

When $a = 4$, $b = \frac{-12}{4} = -3$

When $a = -4$, $b = \frac{-12}{-4} = 3$

$\therefore \sqrt{7 - 24i} = \pm(4 - 3i)$

$\therefore x = \frac{(2+i) \pm (4-3i)}{2}$

$\therefore x = \frac{(2+i) \pm (4-3i)}{2}$ or $x = \frac{(2+i) - (4-3i)}{2}$

$\therefore x = 3 - i$ or $x = -1 + 2i$.

Exercise 3.2 | Q 4.2 | Page 40

Solve the following quadratic equation: $x^2 - \left(3\sqrt{2} + 2i\right)x + 6\sqrt{2}i = 0$

SOLUTION

Given equation is $x^2-\left(3\sqrt{2}+2i\right)x+6\sqrt{2}i = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 1,\ b = -\left(3\sqrt{2}+2i\right), c=6\sqrt{2}i$

Discriminant = $b^2 - 4ac$

$= \left[-\left(3\sqrt{2}+2i\right)\right]^2-4\times1\times6\sqrt{2}i$

$= 18+12\sqrt{2}i+4i^2-24\sqrt{2}i$

$= 18-12\sqrt{2}i-4$ ...[$\because i^2 = -1$]

$= 14-12\sqrt{2}i$

So, the given equation has complex roots.

These roots are given by

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-\left[-\left(3\sqrt{2}+2i\right)\right]\pm\sqrt{14-12\sqrt{2}i}}{2(1)}$

$= \dfrac{\left(3\sqrt{2}+2i\right)\pm\sqrt{14-12\sqrt{2}i}}{2}$

Let $\sqrt{14-12\sqrt{2}i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$14-12\sqrt{2}i = a^2 + i^2b^2 + 2abi$

$\therefore 14-12\sqrt{2}i = (a^2 - b^2) + 2abi$ ...[$\because i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 14$ and $2ab = -12\sqrt{2}$

$\therefore a^2 - b^2 = 14$ and $b = \dfrac{-6\sqrt{2}}{a}$

$\therefore a^2 - \left(\frac{-6\sqrt{2}}{a}\right)^2 = 14$

$\therefore a^2 - \frac{72}{a^2} = 14$

$\therefore a^4 - 72 = 14a^2$

$\therefore a^4 - 14a^2 - 72 = 0$

$\therefore (a^2 - 18)(a^2 + 4) = 0$

$\therefore a^2 = 18$ or $a^2 = -4$

But $a \in R$

$\therefore a^2 \neq -4$

$\therefore a^2 = 18$

$\therefore a = \pm 3\sqrt{2}$

When $a = 3\sqrt{2}, b = \frac{-6\sqrt{2}}{3\sqrt{2}} = -2$

When $a = -3\sqrt{2}, b = \frac{-6\sqrt{2}}{-3\sqrt{2}} = 2$

$\therefore \sqrt{14 - 12\sqrt{2}i} = \pm\left(3\sqrt{2} - 2i\right)$

$\therefore x = \frac{\left(3\sqrt{2} + 2i\right) \pm \left(3\sqrt{2} - 2i\right)}{2}$

$\therefore x = \frac{\left(3\sqrt{2} + 2i\right) + \left(3\sqrt{2} - 2i\right)}{2}$

or $x = \frac{\left(3\sqrt{2} + 2i\right) - \left(3\sqrt{2} - 2i\right)}{2}$

$\therefore x = 3\sqrt{2}$ or $x = 2i$.

Exercise 3.2 | Q 4.3 | Page 40

Solve the following quadratic equation: $x^2 – (5 – 1)x + (18 + i) = 0$

SOLUTION

Given equation is $x^2 – (5 – 1)x + (18 + i) = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1, b = – (5 – i), c = 18 + i$
Discriminant = b2 – 4ac
$= [–(5 – i)]^2 – 4 \times 1 \times (18 + i)$
$= 25 – 10i + i^2 – 72 – 4i$
$= 25 – 10i – 1 – 72 – 4i$ ...[$\because i^2 = – 1$]
$= – 48 – 14i$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$$

$$= \frac{-[-(5 - i)] \pm \sqrt{-48 - 14i}}{2(1)}$$

$$= \frac{(5 - i) \pm \sqrt{-48 - 14i}}{2}$$

Let $\sqrt{-48 - 14i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get
$– 48 – 14i = a^2 + b^2i^2 + 2abi$
$\therefore – 48 – 14i = (a^2 – b^2) + 2abi$ ...[$\because i^2 = – 1$]
Equating real and imaginary parts, we get
$a^2 – b^2 = – 48$ and $2ab = – 14$

$\therefore$ $a^2 - b^2 = -48$ and $b = \dfrac{-7}{a}$

$\therefore$ $a^2 - \left(\dfrac{-7}{a}\right)^2 = -48$

$\therefore$ $a^2 - \dfrac{49}{a^2} = -48$

$\therefore$ $a^4 - 49 = -48a^2$

$\therefore$ $a^4 + 48a^2 - 49 = 0$

$\therefore$ $(a^2 + 49)(a^2 - 1) = 0$

$\therefore$ $a^2 = -49$ or $a^2 = 1$

But $a \in R$

$\therefore$ $a^2 \neq -49$

$\therefore$ $a^2 = 1$

$\therefore$ $a = \pm 1$

When $a = 1$, $b = -\dfrac{7}{1} = -7$

When $a = -1$, $b = \dfrac{-7}{-1} = 7$

$\therefore$ $\sqrt{-48 - 14i} = \pm(1 - 7i)$

$\therefore$ $x = \dfrac{(5 - i) \pm (1 - 7i)}{2}$

$\therefore$ $x = \dfrac{5 - i + 1 - 7i}{2}$ or $x = \dfrac{5 - i - 1 + 7i}{2}$

$\therefore$ $x = 3 - 4I$ or $x = 2 + 3i$.

Exercise 3.2 | Q 4.4 | Page 40

Solve the following quadratic equation: $(2 + i) x^2 - (5 - i) x + 2(1 - i) = 0$

SOLUTION

Given equation is

$(2 + i) x^2 – (5 – i) x + 2(1 – i) = 0$

Comparing with ax2 + bx + c = 0, we geta

$a = 2 + i, b = – (5 – i), c = 2(1 - i)$

Discriminant $= b^2 – 4ac$

$= [– 5 – i)]^2 – 4 x (2 + i) x 2(1 – i)$

$= 25 – 10i + i^2 – 8(2 + i) (1 - i)$

$= 25 – 10i + i^2 – 8(2 – 2i + i – i^2)$

$= 25 – 10i – 1 - 8(2 - i + 1)$ ...[$\because i^2 = – 1$]

$= 25 – 10i – 1 – 16 + 8i – 8$

$= – 2i$

So, the given equation has complex roots.

These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-[-(5-i)] \pm \sqrt{-2i}}{2(2+i)}$$

$$= \frac{(5-i) \pm \sqrt{-2i}}{2(2+i)}$$

Let $\sqrt{-2i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$-2i = a^2 + b^2 i^2 + 2abi$

$\therefore 0 - 2i = (a^2 - b^2) + 2abi$ ...[$\because i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 0$ and $2ab = -2$

$\therefore a^2 - b^2 = 0$ and $b = -\frac{1}{a}$

$\therefore a^2 - \left(\frac{-1}{a}\right)^2 = 0$

$\therefore a^2 - \frac{1}{a^2} = 0$

$\therefore a^4 - 1 = 0$

$\therefore (a^2 - 1)(a^2 + 1) = 0$

$\therefore a^2 = 1$ or $a^2 = -1$

But $a \in R$

$\therefore a^2 \neq -1$

$\therefore a^2 = 1$

$\therefore a = \pm 1$

When $a = 1$, $b = -1$

When $a = -1$, $b = 1$

$\therefore \sqrt{-2i} = \pm (1 - i)$

$$\therefore x = \frac{(5-i) \pm (1-i)}{2(2+i)}$$

$\therefore x = \dfrac{5 - i + 1 - i}{2(2 + i)}$ or $x = \dfrac{5 - i - 1 + i}{2(2 + i)}$

$\therefore x = \dfrac{6 - 2i}{2(2 + i)}$ or $x = \dfrac{4}{2(2 + i)}$

$\therefore x = \dfrac{2(3 - i)}{2(2 + i)}$ or $x = \dfrac{2}{2 + i}$

$\therefore x = \dfrac{3 - i}{2 + i}$ or $x = \dfrac{2(2 - i)}{(2 + i)(2 - i)}$

$\therefore x = \dfrac{(3 - i)(2 - i)}{(2 + i)(2 - i)}$ or $x = \dfrac{2(2 - i)}{4 - i^2}$

$\therefore x = \dfrac{6 - 5i + i^2}{4 - i^2}$ or $x = \dfrac{4 - 2i}{4 - i^2}$

$\therefore x = \dfrac{5 - 5i}{5}$ or $x = \dfrac{4 - 2i}{5}$ ...[$\because i^2 = -1$]

$\therefore x = 1 - i$ or $x = \dfrac{4}{5} - \dfrac{2i}{5}$.

EXERCISE 3.3 [PAGE 42]

Exercise 3.3 | Q 1.1 | Page 42

If ω is a complex cube root of unity, show that $(2 - \omega)(2 - \omega^2) = 7$

SOLUTION

ω is a complex cube root of unity.

$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega, 1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

L.H.S. = $(2 - \omega)(2 - \omega^2)$

= $4 - 2\omega^2 - 2\omega + \omega^3$

= $4 - 2(\omega^2 + \omega) + 1$

= 4 – 2(– 1) + 1

= 4 + 2 + 1

= 7

= R.H.S.

Exercise 3.3 | Q 1.2 | Page 42

If ω is a complex cube root of unity, show that $(2 + \omega + \omega^2)^3 - (1 - 3\omega + \omega^2)^3 = 65$

SOLUTION

ω is a complex cube root of unity.

$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

L.H.S. = $(2 + \omega + \omega^2)^3 - (1 - 3\omega + \omega^2)^3$

= $[2 + (\omega + \omega^2)]^3 - [-3\omega + (1 + \omega^2)]^3$

= $(2 - 1)^3 - (-3\omega - \omega)^3$

= $1^3 - (-4\omega)^3$

= $1 + 64\omega^3$

= 1 + 64(1)

= 65

= R.H.S.

Exercise 3.3 | Q 1.3 | Page 42

If ω is a complex cube root of unity, show that $\frac{(a + b\omega + c\omega^2)}{c + a\omega + b\omega^2} = \omega^2$

SOLUTION

ω is a complex cube root of unity.

$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

L.H.S. = $\dfrac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2}$

= $\dfrac{a\omega^3 + b\omega^4 + c\omega^2}{c + a\omega + b\omega^2}$ $\quad \ldots[\because \omega^3 = 1, \therefore \omega^4 = \omega]$

= $\dfrac{\omega^2(c + a\omega + b\omega^2)}{c + a\omega + b\omega^2}$

= ω^2

= R.H.S.

Exercise 3.3 | Q 2.1 | Page 42

If ω is a complex cube root of unity, find the value of $\omega + \dfrac{1}{\omega}$

SOLUTION

ω is a complex cube root of unity

$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega, 1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$\omega + \dfrac{1}{\omega} = \dfrac{\omega^2 + 1}{\omega} = \dfrac{-\omega}{\omega} = -1.$

Exercise 3.3 | Q 2.2 | Page 42

If ω is a complex cube root of unity, find the value of $\omega^2 + \omega^3 + \omega^4$

SOLUTION

ω is a complex cube root of unity.

$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega, 1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$\omega^2 + \omega^3 + \omega^4$
= $\omega^2\left(1 + \omega + \omega^2\right)$
= $\omega^2(0)$
= 0.

Exercise 3.3 | Q 2.3 | Page 42

If ω is a complex cube root of unity, find the value of $\left(1 + \omega^2\right)^3$

SOLUTION

ω is a complex cube root of unity
$\therefore \omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$

$\left(1 + \omega^2\right)^3 = (-\omega)^3 = -\omega^3 = -1.$

Exercise 3.3 | Q 2.4 | Page 42

If ω is a complex cube root of unity, find the value of $\left(1 - \omega - \omega^2\right)^3 + \left(1 - \omega + \omega^2\right)^3$

SOLUTION

ω is a complex cube root of unity
$\therefore \omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$

$\left(1 - \omega - \omega^2\right)^3 + \left(1 - \omega + \omega^2\right)^3$

= $\left[1 - \left(\omega + \omega^2\right)\right]^3 + \left[\left(1 + \omega^2\right) - \omega\right]^3$

= $[1 - (-1)]^3 + (-\omega - \omega)^3$

= $2^3 + (-2\omega)^3$

= $8 - 8\omega^3$

= $8 - 8(1)$

= 0

Exercise 3.3 | Q 2.5 | Page 42

If ω is a complex cube root of unity, find the value of $(1+\omega)\left(1+\omega^{2}\right)\left(1+\omega^{4}\right)\left(1+\omega^{8}\right)$

SOLUTION

ω is a complex cube root of unity

$\therefore \omega^{3}=1$ and $1+\omega+\omega^{2}=0$

Also, $1+\omega^{2}=-\omega, 1+\omega=-\omega^{2}$ and $\omega+\omega^{2}=-1$

$(1+\omega)\left(1+\omega^{2}\right)\left(1+\omega^{4}\right)\left(1+\omega^{8}\right)$

$=(1+\omega)\left(1+\omega^{2}\right)(1+\omega)\left(1+\omega^{2}\right) \quad \ldots\left[\because \omega^{3}=1, \therefore \omega^{4}=\omega\right]$

$=\left(-\omega^{2}\right)(-\omega)\left(-\omega^{2}\right)(-\omega)$

$=\omega^{6}$

$=\left(\omega^{3}\right)^{2}$

$=(1)^{2}$

$=1.$

Exercise 3.3 | Q 3 | Page 42

If α and β are the complex cube roots of unity, show that $\alpha^2 + \beta^2 + \alpha\beta = 0$.

SOLUTION

α and β are the complex cube roots of unity.

$\therefore \alpha = \dfrac{-1+1\sqrt{3}}{2}$ and $\beta=\dfrac{-1-i\sqrt{3}}{2}$

$\therefore \alpha\beta = \left(\dfrac{-1+i\sqrt{3}}{2}\right)\left(\dfrac{-1-i\sqrt{3}}{2}\right)$

$=\dfrac{(-1)^{2}-\left(i\sqrt{3}\right)^{2}}{4}$

$=\dfrac{1-(-1)(3)}{4} \quad \ldots\left[\because i^{2}=-1\right]$

$= \dfrac{1+3}{4}$

$\therefore$ αβ = 1

Also, α + β = $\dfrac{-1+i\sqrt{3}}{2} + \dfrac{-1-i\sqrt{3}}{2}$

$= \dfrac{-1+i\sqrt{3}-1-i\sqrt{3}}{2}$

$= \dfrac{-2}{2}$

$\therefore$ α + β = − 1

L.H.S. = $\alpha^2 + \beta^2 + \alpha\beta$

= $\alpha^2 + 2\alpha\beta + \beta^2 + \alpha\beta - 2\alpha\beta$ ...[Adding and subtracting 2αβ]

= $(\alpha^2 + 2\alpha\beta + \beta^2) - \alpha\beta$

= $(\alpha + \beta)^2 - \alpha\beta$

= (− 1) − 1

= 0

= R.H.S.

Exercise 3.3 | Q 4 | Page 42

If x = a + b, y = αa + βb and z = aβ + bα, where α and β are the complex cube roots of unity, show that xyz = $a^3 + b^3$.

SOLUTION

x = a + b, y = αa + βb and z = aβ + bα
α and β are the complex cube roots of unity.

$\therefore \alpha = \dfrac{-1+i\sqrt{3}}{2} \text{ and } \beta = \dfrac{-1-i\sqrt{3}}{2}$

$\therefore \alpha\beta = \left(\dfrac{-1+i\sqrt{3}}{2}\right)\left(\dfrac{-1-i\sqrt{3}}{2}\right)$

$= \frac{(-1)^2 - \left(i\sqrt{3}\right)^2}{4}$

$= \frac{1 - (-1)(3)}{4}$ ...[∵ $i^2 = -1$]

$= \frac{1 + 3}{4}$

∴ $\alpha\beta = 1$

Also, $\alpha + \beta = \frac{-1 + i\sqrt{3}}{2} + \frac{-1 - i\sqrt{3}}{2}$

$= \frac{-1 + i\sqrt{3} - 1 - i\sqrt{3}}{2}$

$= \frac{-2}{2}$

∴ $\alpha + \beta = 1$

L.H.S. = xyz = $(a + b)(\alpha a + \beta b)(a\beta + b\alpha)$

$= (a + b)(\alpha\beta a^2 + \alpha^2 ab + \beta^2 ab + \alpha\beta b^2)$

$= (a + b)[1. (a^2) + (\alpha^2 + \beta^2)ab + 1. (b^2)]$

$= (a + b) \{a^2 + [(\alpha + \beta)^2 - 2\alpha\beta]ab + b^2\}$

$= (a + b) \{a^2 + [(-1)^2 - 2(1)]ab + b^2\}$

$= (a + b) [a^2 + (1 - 2)ab + b^2]$

$= (a + b)(a^2 - ab + b^2)$

$= a^3 + b^3$

= R.H.S.

Exercise 3.3 | Q 5.1 | Page 42

If ω is a complex cube root of unity, then prove the following: $\left(\omega^2 + \omega - 1\right)^3 = -8$

SOLUTION

ω is a complex cube root of unity

$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega, 1 + \omega = -\omega^2$

and $\omega + \omega^2 = -1$

L.H.S. = $\left(\omega^2 + \omega - 1\right)^3$

= $(-1 - 1)^3$

= $(-2)^3$

= -8

= R.H.S.

Exercise 3.3 | Q 5.2 | Page 42

If ω is a complex cube root of unity, then prove the following: $(a + b) + \left(a\omega + b\omega^2\right) + \left(a\omega^2 + b\omega\right) = 0$.

SOLUTION

ω is a complex cube root of unity.

$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega, 1 + \omega = -\omega^2$

and $\omega + \omega^2 = -1$

L.H.S. = $(a + b) + \left(a\omega + b\omega^2\right) + \left(a\omega^2 + b\omega\right)$

= $\left(a + a\omega + a\omega^2\right) + \left(b + b\omega + b\omega^2\right)$

= $a\left(1 + \omega + \omega^2\right) + b\left(1 + \omega + \omega^2\right)$

= a(0) + b(0)

= 0

= R.H.S.

MISCELLANEOUS EXERCISE 3 [PAGES 42 - 43]

Miscellaneous Exercise 3 | Q 1 | Page 42

Find the value of $\dfrac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$.

SOLUTION

$$\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$$

$$= \frac{i^{10}\left(i^{582}+i^{580}+i^{578}+i^{576}+i^{574}\right)}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$$

$= i^{10}$

$= (i^4)^2.i^2$

$= (1)^2 (-1)$

$= -1.$

Miscellaneous Exercise 3 | Q 2 | Page 42

Find the value of $\sqrt{-3} \times \sqrt{-6}$.

SOLUTION

$\sqrt{-3} \times \sqrt{-6} = \sqrt{3} \times \sqrt{-1} + \sqrt{6} \times \sqrt{-1}$

$= \sqrt{3}i \times \sqrt{6}i$

$= \sqrt{18}i^2$

$= -3\sqrt{2}$ ...[$\because i^2 = -1$]

Miscellaneous Exercise 3 | Q 3.01 | Page 42

Simplify the following and express in the form a + ib: $3 + \sqrt{-64}$

SOLUTION

$3 + \sqrt{-64}$

$= 3 + \sqrt{64} \cdot \sqrt{-1}$

$= 3 + 8i$

Miscellaneous Exercise 3 | Q 3.02 | Page 42

Simplify the following and express in the form a + ib: $(2i^3)^2$

SOLUTION

$(2i^3)^2 = 4i^6$
$= 4(i^2)^3$
$= 4(-1)^3$ ...[$\because i^2 = -1$]
$= -4$
$= -4 + 0i$

Miscellaneous Exercise 3 | Q 3.03 | Page 42
Simplify the following and express in the form a + ib: $(2 + 3i)(1 - 4i)$

SOLUTION

$(2 + 3i)(1 - 4i)$
$= 2 - 8i + 3i - 12i^2$
$= 2 - 5i - 12(-1)$ ...[$\because i^2 = -1$]
$= 14 - 5i$

Miscellaneous Exercise 3 | Q 3.04 | Page 42

Simplify the following and express in the form a + ib: $\frac{5}{2}i(-4 - 3i)$

SOLUTION

$$\frac{5}{2}i(-4 - 3i)$$
$$= \frac{5}{2}i\left(-4i - 3i^2\right)$$
$$= \frac{5}{2}[-4i - 3(-1)] \quad ...[\because i^2 = -1]$$
$$= \frac{5}{2}(3 - 4i)$$
$$= \frac{15}{2} - 10i$$

Miscellaneous Exercise 3 | Q 3.05 | Page 42
Simplify the following and express in the form a + ib: $(1 + 3i)^2 (3 + i)$

SOLUTION

$(1 + 3i)^2 (3 + i)$
$= (1 + 6i + 9i^2)(3 + i)$
$= (1 + 6i - 9)(3 + i)$ ...[$\because i^2 = -1$]

$= (-8 + 6i)(3 + i)$
$= -24 - 8i + 18i + 6i^2$
$= -24 + 10i + 6(-1)$
$= -24 + 10i - 6$
$= -30 + 10i$

Miscellaneous Exercise 3 | Q 3.06 | Page 42

Simplify the following and express in the form a + ib: $\frac{4 + 3i}{1 - i}$

SOLUTION

$$\frac{4 + 3i}{1 - i} = \frac{(4 + 3i)(1 + i)}{(1 - i)(1 + i)}$$

$$= \frac{4 + 4i + 3i + 3i^2}{1 - i^2}$$

$$= \frac{4 + 7i + 3(-1)}{1 - (-1)} \quad ...[\because i^2 = -1]$$

$$= \frac{1 + 7i}{2}$$

$$= \frac{1}{2} + \frac{7}{2}i.$$

Miscellaneous Exercise 3 | Q 3.07 | Page 42

Simplify the following and express in the form a + ib: $\left(1 + \frac{2}{i}\right)\left(3 + \frac{4}{i}\right)(5 + i)^{-1}$

SOLUTION

$$\left(1+\frac{2}{i}\right)\left(3+\frac{4}{i}\right)(5+i)^{-1}$$

$$= \frac{(i+2)}{i}\cdot\frac{(3i+4)}{i}\cdot\frac{1}{5+i}$$

$$= \frac{3i^2\ 4i+6i+8}{i^2(5+i)}$$

$$= \frac{-3+10i+8}{-1(5+i)} \quad ...[\because i^2 = -1]$$

$$= \frac{(5+10i)}{-(5+i)}$$

$$= \frac{(5+10i)(5-i)}{-(4+i)(5-i)}$$

$$= \frac{25-5i+50i-10i^2}{-(25-i^2)}$$

$$= \frac{25+45i-10(-1)}{-[25-(-1)]}$$

$$= \frac{35+45i}{-26}$$

$$= \frac{-35}{26}-\frac{45}{26}i$$

Miscellaneous Exercise 3 | Q 3.08 | Page 42

Simplify the following and express in the form a + ib: $\frac{\sqrt{5}+\sqrt{3}i}{\sqrt{5}-\sqrt{3}i}$

SOLUTION

$$\frac{\sqrt{5}+\sqrt{3}\mathrm{i}}{\sqrt{5}-\sqrt{3}\mathrm{i}}$$

$$= \frac{\left(\sqrt{5}+\sqrt{3}\mathrm{i}\right)\left(\sqrt{5}+\sqrt{3}\mathrm{i}\right)}{\left(\sqrt{5}-\sqrt{3}\mathrm{i}\right)\left(\sqrt{5}+\sqrt{3}\mathrm{i}\right)}$$

$$= \frac{5+2\sqrt{15}\mathrm{i}+3\mathrm{i}^2}{5-3\mathrm{i}^2}$$

$$= \frac{5+2\sqrt{15}\mathrm{i}+3(-1)}{5-3(-1)}$$ ...[∵ $i^2 = -1$]

$$= \frac{2+2\sqrt{15}\mathrm{i}}{8}$$

$$= \frac{1+\sqrt{15}\mathrm{i}}{4}$$

$$= \frac{1}{4}+\frac{\sqrt{15}\mathrm{i}}{4}.$$

Miscellaneous Exercise 3 | Q 3.09 | Page 43

Simplify the following and express in the form a + ib: $\frac{3\mathrm{i}^5+2\mathrm{i}^7+\mathrm{i}^9}{\mathrm{i}^6+2\mathrm{i}^8+3\mathrm{i}^{18}}$

SOLUTION

$$\frac{3\mathrm{i}^5+2\mathrm{i}^7+\mathrm{i}^9}{\mathrm{i}^6+2\mathrm{i}^8+3\mathrm{i}^{18}}$$

$$= \frac{3\left(\mathrm{i}^4\cdot\mathrm{i}\right)2\left(\mathrm{i}^4\cdot\mathrm{i}^3\right)+\left(\mathrm{i}^4\right)^2\cdot\mathrm{i}}{\mathrm{i}^4\cdot\mathrm{i}^2+2\left(\mathrm{i}^4\right)+3\left(\mathrm{i}^2\right)^9}$$

$$= \frac{3(1)\cdot\mathrm{i}+2(1)(-\mathrm{i})+(1)^2\cdot\mathrm{i}}{(1)(-1)+2(1)^2+3(-1)^9}$$ ...[∵ $i^2 = -1$, $i^3 = -i$, $i^4 = 1$]

$= \dfrac{3i - 2i + i}{-1 + 2 - 3}$

$= \dfrac{2i}{-2}$

= – i

= 0 – i

Miscellaneous Exercise 3 | Q 3.1 | Page 43

Simplify the following and express in the form a + ib: $\dfrac{5+7i}{4+3i} + \dfrac{5+7i}{4-3i}$

SOLUTION

$\dfrac{5+7i}{4+3i} + \dfrac{5+7i}{4-3i}$

$= (5+7i)\left[\dfrac{1}{4+3i} + \dfrac{1}{4-3i}\right]$

$= (5+7i)\left[\dfrac{4-3i+4+3i}{(4+3i)(4-3i)}\right]$

$= (5+7i)\left[\dfrac{8}{16-9i^2}\right]$

$= (5+7i)\left[\dfrac{8}{16-9(-1)}\right]$ ...[∵ $i^2 = -1$]

$= \dfrac{8(5+7i)}{25}$

$= \dfrac{40+56i}{25}$

$= \dfrac{40}{25} + \dfrac{56}{25}i$

$= \dfrac{8}{5} + \dfrac{56}{25}i.$

Miscellaneous Exercise 3 | Q 4.1 | Page 43

Solve the following equation for x, y ∈ R: (4 – 5i) x + (2 + 3i) y = 10 – 7i

SOLUTION

(4 – 5i) x + (2 + 3i) y = 10 – 7i
∴ (4x + 2y) + (3y - 5x) i = 10 – 7i
Equating real and imaginary parts, we get
4x + 2y = 10
i.e., 2x + y = 5 ...(i)
and 3y – 5x = – 7 ...(ii)
Equation (i) x 3 – equation (ii) gives
11x = 22
∴ x = 2
Putting x = 2 in (i), we get
2(2) + y = 5
∴ y = 1
∴ x = 2 and y = 1.

Miscellaneous Exercise 3 | Q 4.2 | Page 43

Solve the following equation for x, y ∈ R: (1 – 3i) x + (2 + 5i) y = 7 + i

SOLUTION

(1 – 3i) x + (2 + 5i) y = 7 + i
∴ (x + 2y) + (– 3x + 5y)i = 7 + i
Equating real and imaginary parts, we get
x + 2y = 7 ...(i)
and – 3x + 5y = 1 ...(ii)
Equation (i) x 3 + equation (ii) gives
11y = 22
∴ y = 2
Putting y = 2 in (i), we get
x + 2(2) = 7
∴ x = 3
∴ x = 3 and y = 2.

Miscellaneous Exercise 3 | Q 4.3 | Page 43

Solve the following equation for x, y ∈ R: $\frac{x + iy}{2 + 3i} = 7 - i$

SOLUTION

$\frac{x + iy}{2 + 3i}$ = 7 – i

∴ x + iy = (7 – i)(2 + 3i)

∴ x + iy = 14 + 21i – 2i – $3i^2$

∴ x + iy = 14 + 19i – 3(– 1) ...[∵ i^2 = – 1]

∴ x + iy = 17 + 19i

Equating real and imaginary parts, we get

x = 17 and y = 19

Miscellaneous Exercise 3 | Q 4.4 | Page 43

Solve the following equation for x, y ∈ R: (x + iy)(5 + 6i) = 2 + 3i

SOLUTION

(x + iy)(5 + 6i) = 2 + 3i

∴ x + iy = $\frac{2 + 3i}{5 + 6i}$

∴ x + iy = $\frac{(2 + 3i)(5 - 6i)}{(5 + 6i)(5 - 6i)}$

= $\frac{10 - 12i + 15i - 18i^2}{25 - 36i^2}$ ...[∵ i^2 = – 1]

= $\frac{10 + 3i - 18(-1)}{25 - 36(-1)}$

∴ x + iy = $\frac{28 + 3i}{61}$

= $\frac{28}{61} + \frac{3}{61}i$

Equating real and imaginary parts, we get

x = $\frac{28}{61}$ and $y = \frac{3}{61}$.

Miscellaneous Exercise 3 | Q 4.5 | Page 43

Solve the following equation for x, y ∈ R: $2x + i^9 y (2 + i) = x i^7 + 10 i^{16}$

SOLUTION

$2x + i^9 y (2 + i) = x i^7 + 10 i^{16}$

$\therefore 2x + (i^4)^2.i.y (2 + i) = x (i^2)^3.i + 10.(i^4)^4$

$\therefore 2x + (1)^2.iy (2 + i) = x (-1)^3.i + 10 (1)^4$ ...[$\because i^2 = -1, i^4 = 1$]

$\therefore 2x + 2yi + yi^2 = -xi + 10$

$\therefore 2x + 2yi - y + xi = 10$

$\therefore (2x - y) + (x + 2y)i = 10 + 0.i$

Equating real and imaginary parts, we get

$2x - y = 10$ ...(i)

and $x + 2y = 0$ ...(ii)

Equation (i) x 2 + equation (ii) gives

$5x = 20$

$\therefore x = 4$

Putting x = 4 in (i), we get

$2(4) - y = 10$

$\therefore y = 8 - 10$

$\therefore y = -2$

$\therefore x = 4$ and $y = -2$

Miscellaneous Exercise 3 | Q 5.1 | Page 43

Find the value of : $x^3 + 2x^2 - 3x + 21$, if $x = 1 + 2i$

SOLUTION

$x = 1 + 2i$

$\therefore x - 1 = 2i$

$\therefore (x - 1)^2 = 4i^2$

$\therefore x^2 - 2x + 1 = -4$ ...[$\because i^2 = -1$]

$\therefore x^2 - 2x + 5 = 0$ ...(i)

$$\because\ x^2 - 2x + 5 \overline{)\, x^3 + 2x^2 - 3x + 21\,}\quad x + 4$$

$$\begin{array}{r} x^3 - 2x^2 + 5x \\ - \quad + \quad - \\ \hline 4x^2 - 8x + 21 \\ 4x^2 - 8x + 20 \\ - \quad + \quad - \\ \hline 1 \end{array}$$

$\therefore x^3 + 2x^2 - 3x + 21$

$= (x^2 - 2x + 5)(x + 4) + 1$

$= 0.(x + 4) + 1$ ...[From (i)]

$= 0 + 1$

$\therefore x^3 + 2x^2 - 3x + 21 = 1$

Miscellaneous Exercise 3 | Q 5.2 | Page 43

Find the value of : $x^3 - 5x^2 + 4x + 8$, if $x = \frac{10}{3 - i}$

SOLUTION

$x = \frac{10}{3 - i}$

$\therefore x = \frac{10(3 + i)}{(3 - i)(3 + i)}$

$= \frac{10(3 + i)}{9 - i^2}$

$= \dfrac{10(3+i)}{9-(-1)}$ ...[∵ $i^2 = -1$]

$= \dfrac{10(3+i)}{10}$

∴ $x = 3 + i$

∴ $x - 3 = i$

∴ $(x - 3)^2 = i^2$

∴ $x^2 - 6x + 9 = -1$ ...[∵ $i^2 = -1$]

∴ $x^2 - 6x + 10 = 0$ ...(i)

$$x + 1$$

$$x^2 - 6x + 10)\overline{x^3 + 5x^2 + 4x + 8}$$

$$x^3 - 6x^2 + 10x$$

$$- \quad + \quad -$$

$$x^2 - 6x + 8$$

$$x^2 - 6x + 10$$

$$- \quad + \quad -$$

$$-2$$

∴ $x^3 - 5x^2 + 4x + 8$

$= (x^2 - 6x + 10)(x + 1) - 2$

$= 0.(x + 1) - 2$ ...[From (i)]

$= 0 - 2$

∴ $x^3 - 5x^2 + 4x + 8 = -2.$

Miscellaneous Exercise 3 | Q 5.3 | Page 43

Find the value of: $x^3 - 3x^2 + 19x - 20$, if $x = 1 - 4i$

SOLUTION

$x = 1 - 4i$

∴ $x - 1 = -4i$

$\therefore (x - 1)^2 \;\; 16i^2$

$\therefore x^2 - 2x + 1 = -16$ ...[∵ $i^2 = -1$]

$\therefore x^2 - 2x + 17 = 0$ ...(i)

$$\begin{array}{r|l} & x - 1 \\ \hline x^2 - 2x + 17 & x^3 - 3x^2 + 19x^2 - 20 \\ & x^3 - 2x^2 + 17x \\ & - \quad + \quad - \\ \hline & -x^2 + 2x - 20 \\ & -x^2 + 2x - 17x \\ & - \quad + \quad - \\ \hline & -3 \end{array}$$

$\therefore x^3 - 3x^2 + 19x - 20$

$= (x^2 - 2x + 17)(x - 1) - 3$

$= 0.(x - 1) - 3$ ...[From (i)]

$= 0 - 3$

$\therefore x^3 - 3x^2 + 19x - 20 = -3$

Miscellaneous Exercise 3 | Q 6.1 | Page 43

Find the square root of: $-16 + 30i$

SOLUTION

Let $\sqrt{-16 + 30i} = a + bi$, where a, b ∈ R

Squaring on both sides, we get

$-16 + 30i = a^2 + b^2i^2 + 2abi$

$\therefore -16 + 30i = (a^2 - b^2) + 2abi$ ...[∵ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = -16$ and $2ab = 30$

$\therefore a^2 - b^2 = -16$ and $b = \frac{15}{a}$

$\therefore a^2 - \frac{225}{a^2} = -16$

$\therefore a^4 - 225 - 16a^2$

$\therefore a^4 + 16a^2 - 225 = 0$

$\therefore (a^2 + 25)(a^2 - 9) = 0$

$\therefore a^2 = -25$ or $a^2 = 9$

But $a \in R$

$\therefore a^2 \neq -25$

$\therefore a^2 = 9$

$\therefore a = \pm 3$

When $a = 3$, $b = \frac{15}{3} = 5$

When $a = -3$, $b = \frac{15}{-3} = -5$

$\therefore \sqrt{-16 + 30i} = \pm (3 + 5i)$

Miscellaneous Exercise 3 | Q 6.2 | Page 43

Find the square root of 15 – 8i

SOLUTION

Let $\sqrt{15 - 8i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$15 - 8i = a^2 b^2 i^2 + 2abi$

$\therefore 15 - 8i = (a^2 - b^2) + 2abi$ ...[$\because i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 15$ and $2ab = -8$

$\therefore a^2 - b^2 = 15$ and $b = \dfrac{-4}{a}$

$\therefore a^2 \left(\dfrac{-4}{a}\right)^2 = 15$

$\therefore a^2 - \dfrac{16}{a^2} = 15$

$\therefore a^4 - 16 = 15a^2$

$\therefore a^4 - 15a^2 - 16 = 0$

$\therefore (a^2 - 16)(a^2 + 1) = 0$

$\therefore a^2 = 16$ or $a^2 = -1$

But $a \in R$

$\therefore a^2 \neq -1$

$\therefore a^2 = 16$

$\therefore a = \pm 4$

When $a = 4$, $b = \dfrac{-4}{4} = -1$

$\therefore \sqrt{15 - 8i} = \pm (4 - i)$.

Miscellaneous Exercise 3 | Q 6.3 | Page 43

Find the square root of: $2 + 2\sqrt{3}i$

SOLUTION

Let $\sqrt{2 + 2\sqrt{3}i} = a + bi$, where $a, b \in R$.

Squaring on both sides, we get

$\mathbf{2 + 2\sqrt{3}i}$ = $a^2 + b^2i^2 + 2abi$

$\therefore$ $\mathbf{2 + 2\sqrt{3}i}$ = $a^2 - b^2 + 2abi$ ...[$\because$ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 2$ and $2ab = \mathbf{2\sqrt{3}}$

$\therefore$ $a^2 - b^2 = 2$ and $b = \frac{\sqrt{3}}{a}$

$\therefore a^2 - \left(\frac{\sqrt{3}}{a}\right)^2 = 2$

$\therefore a^2 - \frac{3}{a^2} = 2$

$\therefore a^4 - 3 = 2a^2$

$\therefore a^4 - 2a^2 - 3 = 0$

$\therefore (a^2 - 3)(a^2 + 1) = 0$

$\therefore a^2 = 3$ or $a^2 = -1$

But $a \in R$

$\therefore a^2 \neq -1$

$\therefore a^2 = 3$

$\therefore a = \pm \sqrt{3}$

When $a = \sqrt{3}, b = \frac{\sqrt{3}}{\sqrt{3}} = 1$

When $a = -\sqrt{3}, b = \frac{\sqrt{3}}{-\sqrt{3}} = -1$

$\therefore \sqrt{2 + 2\sqrt{3}i} = \pm\left(\sqrt{3} + i\right)$.

Miscellaneous Exercise 3 | Q 6.4 | Page 43

Find the square root of : 18i

SOLUTION

Let $\sqrt{18i}$ = a + bi, where a, b ∈ R

Squaring on both sides, we get

$18i = a^2 + b^2i^2 + 2abi$

$\therefore 0 + 18i = a^2 - b^2 + 2abi$ ...[∵ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 0$ and $2ab = 18$

$\therefore a^2 - b^2 = 0$ and $b = \frac{9}{a}$

$\therefore a^2 - \left(\frac{9}{a}\right)^2 = 0$

$\therefore a^2 - \frac{81}{a^2} = 0$

$\therefore a^4 - 81 = 0$

$\therefore (a^2 - 9)(a^2 + 9) = 0$

$\therefore a^2 = 9$ or $a^2 = -9$

But a ∈ R

$\therefore a^2 \neq -9$

$\therefore a^2 = 9$

$\therefore a = \pm 3$

When $a = 3$, $b = \frac{9}{3} = 3$

When $a = -3$, $b = \frac{9}{-3} = -3$

$\therefore \sqrt{18i} = \pm(3 + 3i) = \pm 3(1 + i)$.

Miscellaneous Exercise 3 | Q 6.5 | Page 43

Find the square root of: 3 – 4i

SOLUTION

Let $\sqrt{3-4i}$ = a + bi, where a, b ∈ R

Squaring on both sides, we get

$3 - 4i = a^2 + b^2i^2 + 2abi$

$\therefore 3 - 4i = a^2 - b^2 + 2abi$ ...[$\because i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 3$ and $2ab = -4$

$\therefore a^2 - b^2 = 3$ and $b = \frac{-2}{a}$

$\therefore a^2 - \left(-\frac{2}{a}\right)^2 = 3$

$\therefore a^2 - \frac{4}{a^2} = 3$

$\therefore a^4 - 4 = 3a^2$

$\therefore a^4 - 3a^2 - 4 = 0$

$\therefore (a^2 - 4)(a^2 + 1) = 0$

$\therefore a^2 = 4$ or $a^2 = -1$

But a ∈ R

$\therefore a^2 \neq -1$

$\therefore a^2 = 4$

$\therefore a = \pm 2$

When a = 2, $b = \frac{-2}{2} = -1$

When a = − 2, $b = \frac{-2}{-2} = 1$

$\therefore \sqrt{3-4i} = \pm (2 - i)$.

Miscellaneous Exercise 3 | Q 6.6 | Page 43

Find the square root of: 6 + 8i

SOLUTION

Let $\sqrt{6+8i}$ = a + bi, where a, b ∈ R

Squaring on both sides, we get

$6 + 8i = a^2 + b^2i^2 + 2abi$

$\therefore 6 + 8i = a^2 - b^2 + 2abi$ ...[$\because i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 6$ and $2ab = 8$

$\therefore a^2 - b^2 = 6$ and $b = \frac{4}{a}$

$\therefore a^2 - \left(\frac{4}{a}\right)^2 = 6$

$\therefore a^2 - \frac{16}{a^2} = 6$

$\therefore a^4 - 16 = 6a^2$

$\therefore a^4 - 6a^2 - 16 = 0$

$\therefore (a^2 - 8)(a^2 + 2) = 0$

$\therefore a^2 = 8$ or $a^2 = -2$

But

$\therefore a^2 \neq -2$

$\therefore a^2 = 8$

$\therefore a = \pm 2\sqrt{2}$

When $a = 2\sqrt{2},\ b = \frac{4}{2\sqrt{2}} = \sqrt{2}$

When a = $-2\sqrt{2}$, $b = \dfrac{4}{-2\sqrt{2}} = -\sqrt{2}$

$\therefore \sqrt{6+8i} = \pm\left(2\sqrt{2}+\sqrt{2}i\right) = \pm\sqrt{2}(2+i).$